Applications of Laplace's law in clinical medicine: a radiological pictorial review

Laplace's law describes the relationship between wall tension, thickness, pressure and radius in a tubular structure. This article provides radiological examples of disease and uses Laplace's law to explain the rationale behind their screening, early diagnosis and treatment.

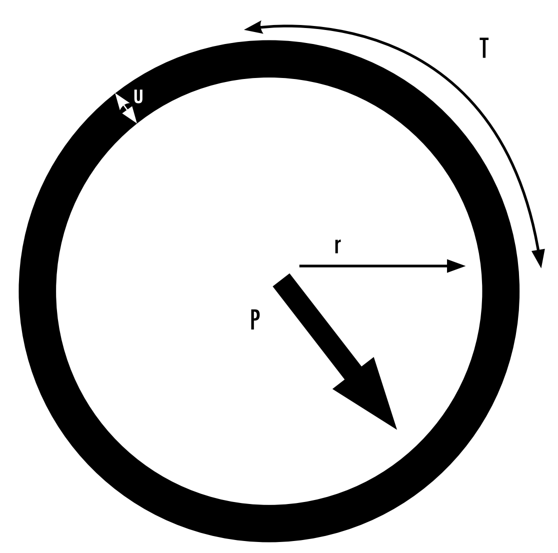
Laplace's law states that within a tubular structure, the transmural pressure required to cause distension is proportional to the wall thickness and surface wall tension, and inversely proportional to its radius. In other words, a structure with a larger radius and low surface tension requires less change in pressure to cause further dilatation than one with a smaller radius and high surface tension (Basford, 2002) (*Figure 1*).

To visualize this in action, imagine inflating a spherical balloon. The initial breaths required to distend the balloon are difficult as the radius of the balloon is small and the surface tension high, but as the balloon is inflated and the radius increases, less pressure is required to distend its walls.

This theory is most widely credited to the 18th century French mathematician, Pierre-Simon Laplace, and it shall be referred to as Laplace's law here. Nonetheless, controversy does exist regarding its origins and some historians claim the concept was initially established a century earlier by the Bernoulli family (Basford, 2002).

The following pictorial review presents images demonstrating the relevance and importance of Laplace's law in clinical medicine.

Figure 1. Diagrammatic explanation and equation for Laplace's law.



$$\text{Transmural pressure (P)} = \frac{2 \times \text{wall thickness (2u)} \times \text{wall tension (T)}}{\text{Radius (r)}}$$

Applications in clinical medicine

Vascular aneurysms

Vascular aneurysms encompass a vast range of aetiologies and occur in various locations but the risk of rupture is common to all (Mower et al, 1993). According to Laplace's law, as the radius of the aneurysm grows, the pressure required to further distend it decreases. In practice, this tells us that the greater the diameter of the aneurysm, the higher the risk of rupture.

The risk of rupture of abdominal aortic aneurysms is estimated at less than 5% per year for aneurysms under 5 cm in cross-sectional diameter, more than doubling to 10–20% per year if the cross-sectional diameter is 6–7 cm (Brewster et al, 2003). For this reason, patients with aneurysms >5.5 cm found on screening are referred for further management (Lindholt et al, 2010) (*Figures 2* and *3*).

Figure 2. Axial contrast-enhanced (porto-venous phase) computed tomography image demonstrating a large abdominal aortic aneurysm.

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Similarly patients who are found to suffer from cerebral vascular aneurysms are referred for neurosurgical opinion if symptomatic or measuring above a critical level.

Congestive cardiac failure

Cardiomegaly (*Figure 4*), a hallmark of congestive cardiac failure on plain film, is caused by increasing left ventricular volume and hypertrophy. In the initial stages there is reduced contractility of the left ventricle with decreased stroke volume and resultant left ventricular dilatation (Tonnessen and Knudson, 2005). According to Laplace, for a certain pressure, an increased chamber radius requires a greater wall thickness to maintain wall tension (Basford, 2002) – hence muscular hypertrophy.

With a thickened (and stiffer) left ventricle, compliance is reduced and filling of the chamber becomes less effective. There is reduced ejection fraction with each beat and further ventricular dilatation.

Toxic megacolon

Toxic megacolon is non-obstructive dilatation of the colon to at least 6 cm (*Figure 5*), with associated systemic toxicity. It can occur secondary to inflammatory bowel disease or infectious colitis, increasingly in association with pseudomembranous colitis. Inflammation extends beyond the colonic mucosa to the smooth muscle layer, causing paralysis and an inability to maintain wall tension. Perforation may result from the unopposed increase in bowel radius and is an indication for surgery (Sheth and LaMont, 1998).

Giant diverticulum

Giant diverticula have been reported measuring up to 30 cm in size. Over 90% arise from the sigmoid colon.

Figure 3. Sagittal reconstruction of a contrast-enhanced (arterial phase) computed tomography scan demonstrating an infrarenal abdominal aortic aneurysm.

Figure 4. An erect chest radiograph demonstrating massive cardiomegaly in the setting of dilated cardiomyopathy.

Figure 5. Plain abdominal radiograph demonstrating toxic megacolon in a patient with ulcerative colitis. Note the colonic dilatation with thumbprinting – indicative of mucosal oedema.

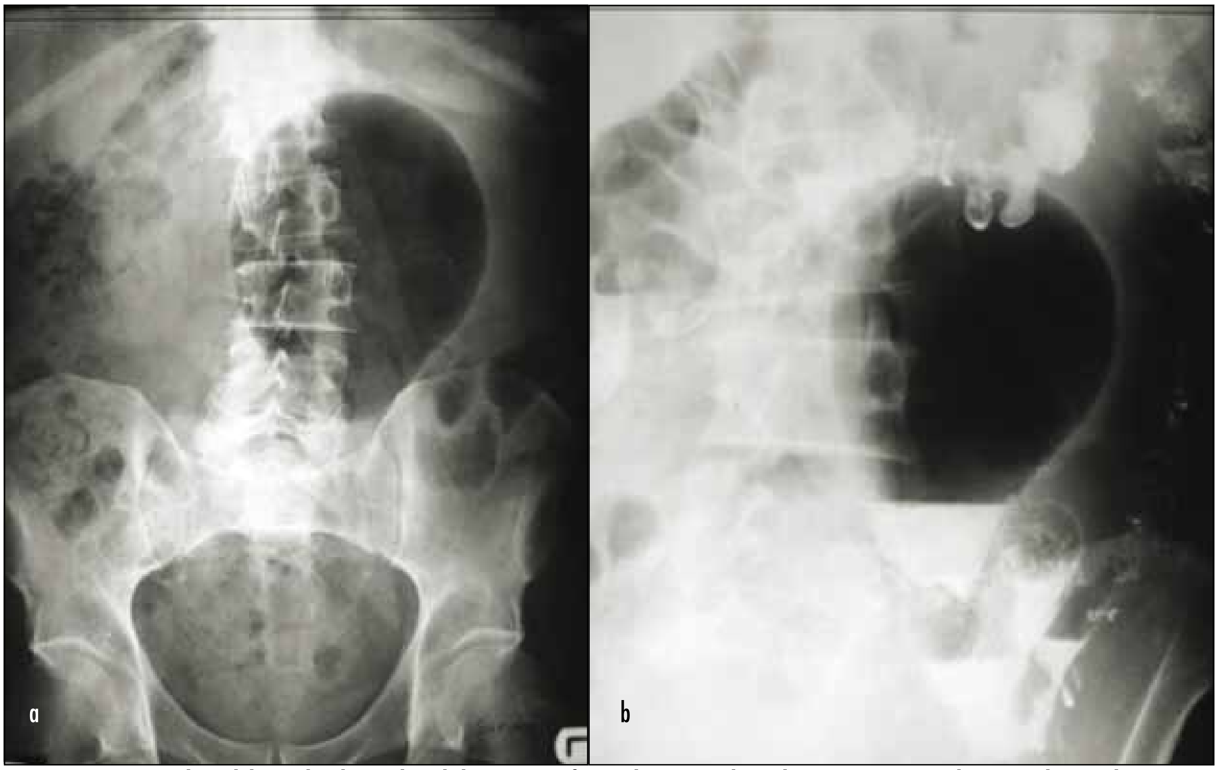


Figure 6. a. Supine plain abdominal radiograph and (b) an image from a barium study in the same patient with a giant diverticulum arising from the sigmoid colon demonstrating an air-barium level.

Most patients have background diverticular disease. Three subtypes have been described: the most common are inflammatory diverticula secondary to a contained perforation that grows as a result of a ball-valve mechanism or gas-forming organisms (*Figure 6*). Complications include perforation and abscess formation. The definitive treatment is surgery (Thomas et al, 2006).

Acute gastric dilatation

Cases have been reported in association with trauma, diabetic ketoacidosis, sepsis, opiate overdose and superior mesenteric artery syndrome. As the stomach dilates, intragastric pressures may exceed venous pressure, resulting in decreased intramural blood flow with mural necrosis and eventually perforation (*Figure 7*). Early gastric decompression may prevent the increasing radius, decreased wall thickness and spiralling wall tension predicted by Laplace's law (Luncă et al, 2005).

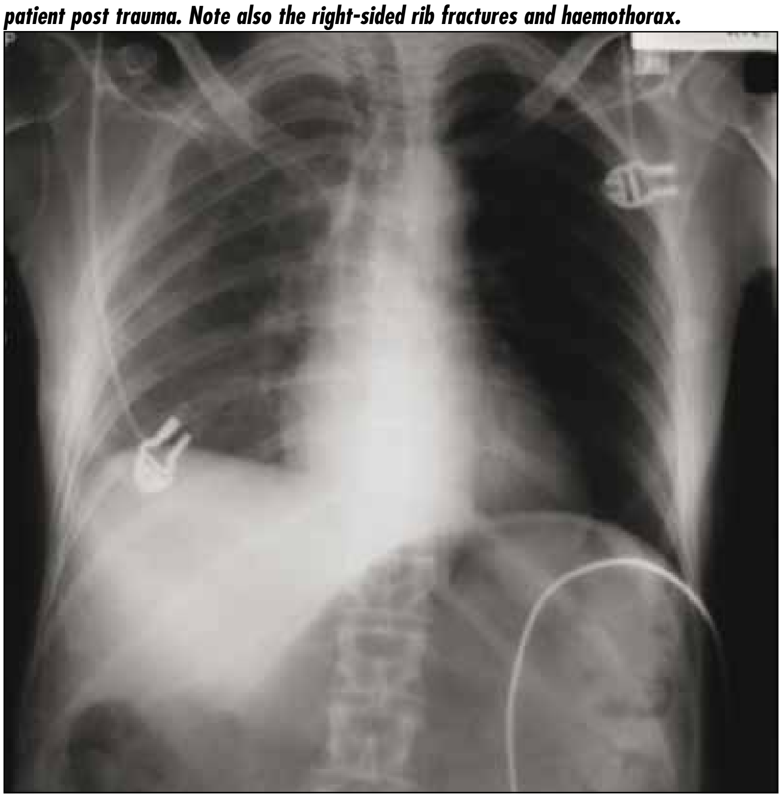
Closed loop obstruction

According to Laplace, as the diameter of a hollow viscus increases in an obstructed segment of bowel, there is a reduction in the transmural pressure required to overcome the surface tension of the wall (*Figures 8–10*). This places it at risk of perforation. In the setting of a sigmoid volvulus, insertion of a flatus tube usually prompts rapid decompression of the bowel and a return to a more normal calibre. If obstruction persists, surgery may reduce mortality from ischaemia and perforation (Levsky et al, 2010).

Pulmonary bullae

Pulmonary bullae are thin-walled air-containing structures measuring over 1 cm. They may be congenital or

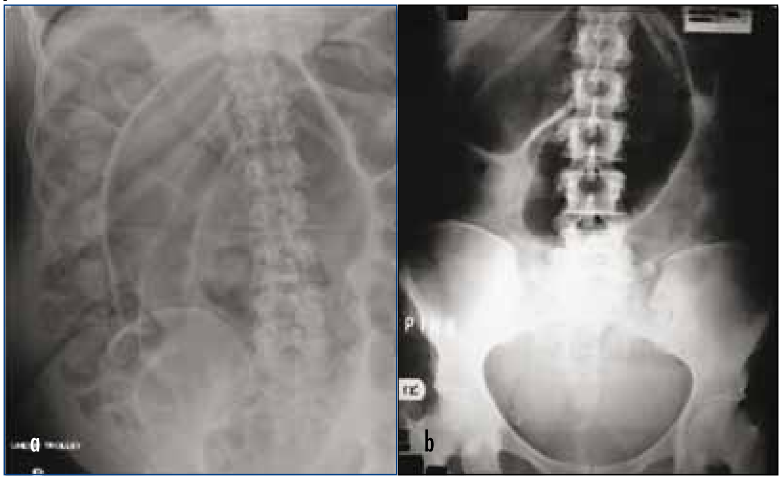
Figure 7. A plain chest radiograph demonstrating massive gastric dilatation in an elderly patient post trauma. Note also the right-sided rib fractures and haemothorax.



acquired in respiratory disease (e.g. cystic lung diseases or bullous emphysema). These air spaces can be prone to infection or rupture leading to pneumothorax. Laplace's law dictates that larger bullae require less transmural pressure to overcome surface tension with increasing likelihood of spontaneous pneumothoraces. Patients with pulmonary bullae are generally asymptomatic and the majority are detected incidentally (*Figures 11* and *12*). Studies to prove that larger bullae are more likely to rupture are lacking and the relationship can only be assumed. Symptomatic patients, suffering shortness of breath or compression of normal lung tissue, may be offered elective bullectomy (Mitlehner et al, 1992).

Figure 8. Axial contrast-enhanced computed tomography image of the lower abdomen demonstrates several loops of dilated bowel suggestive of obstruction. A 'whirl' sign of the mesenteric vessels seen centrally is characteristic of a volvulus.

Figure 9. a. Plain abdominal radiograph demonstrating a sigmoid volvulus. Note the large loop of large bowel seen centrally which demonstrates the so-called 'coffee bean' sign (indicative of the volvulus) as well as multiple dilated loops of large bowel proximal to this reflecting the ensuing large bowel obstruction. b. Plain abdominal radiograph demonstrating a caecal volvulus. Note again the dilated loops of large bowel within the centre of the film, but with proximally dilated small loops and paucity of gas within the pelvis.

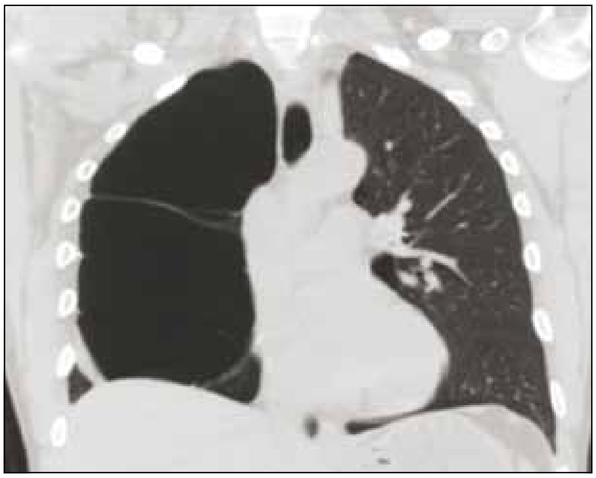


Respiratory distress syndrome

This medical condition presents a different view of Laplace's law. Alveoli have very small radii, meaning that a large transmural pressure is required to overcome the high surface tension. In healthy subjects, pulmonary sur-

Figure 10. Plain abdominal radiograph demonstrating a small bowel obstruction. Note the multiple dilated loops of small bowel within the centre of the abdomen with a paucity of bowel gas in the pelvis and lack of large bowel dilatation. A large calcific mass projected over the sacrum, with radiographic evidence of pneumoperitoneum is suggestive for the diagnosis of gallstone ileus. Incidental note is made of a right fractured neck of femur and a left femoral prosthesis.

Figure 11. Coronal reformatted computed tomography image of the thorax demonstrating at least two large right-sided pulmonary bullae. The mediastinum is not shifted and the bullae appear to displace normal right lung inferiorly. A perceptible thin line separating the two bullae represents their wall.



factant lines alveoli reducing their surface tension, keeping them expanded, even during expiration.

In respiratory distress syndrome, premature infants lack this surfactant and experience greater than normal surface tensions on alveolar walls. The alveoli collapse during expiration, and the pressure required to re-inflate them cannot be achieved. The resultant appearance on radiographs is of widespread patchy airspace opacification representing collapsed alveoli (*Figure 13*). The greater work of breathing leads to exhaustion and neonatal death.

Current treatment methods aim to reduce the surface tension of the alveoli. Intra-tracheal administration of synthetic surfactant reduces alveolar surface tension and subsequently the risk of chronic lung disease, interstitial emphysema and neonatal mortality (Soll, 1998).

Figure 12. Axial computed tomography image (lung windows) demonstrating a large right anterior apical pulmonary bulla. Note the absence of lung markings which suggest an air-filled structure with a perceptible thin wall.

Figure 13. Plain film radiograph of a neonate demonstrating the characteristic features of neonatal respiratory distress syndrome namely poorly inflated lungs with widespread bilateral ground glass opacification.

Urinary retention with spontaneous bladder rupture

Urinary retention is commonly seen within the context of benign prostatic hyperplasia although other causes include drug therapy and neurological impairment. Obstruction to urinary outflow results in a grossly dilated bladder and potential renal failure from hydronephrosis or, rarely, bladder rupture. By applying Laplace's law, the more distended the bladder, the greater ease with which it can expand (*Figure 14*). Immediate catheterization reducing intravesical pressure is crucial.

When micturating, we also know from Laplace's law that the pressure exerted by the bladder wall to ensure a continuous stream of urine during micturition is greater when the bladder is not fully expanded (Basford, 2002). Repeated overexpansion of the bladder over time can lead to long-term complications such as renal failure from hydronephrosis and reflux, bladder diverticula and in extreme cases bladder atony.

Intervertebral disc herniation

One unusual factor that has been shown to contribute to lumbar intervertebral disc herniation is disc shape with more circular endplates resulting in a greater likelihood of posterior herniation (Harrington et al, 2001). It is

Figure 14. Axial contrast-enhanced (porto-venous phase) computed tomography image of the pelvis demonstrating an extremely distended urinary bladder resulting from an enlarged prostate.

thought that the disc exhibits properties of a fluid-filled space (like a blood vessel) and the more circular endplate shape results in a longer anteroposterior radius with resultant higher surface tension of the disc. This increased surface tension is presumed to increase the likelihood of herniation.

Conclusions

Laplace's law is widely applicable in many areas of clinical medicine not only in understanding disease processes but also in the most appropriate steps for their management. Several of these disease processes and their imaging findings have been reviewed. **BJHM**

KEY POINTS

- Laplace's law describes how the pressure required to cause a tubular structure decreases at it gets larger, but increases if the wall is thicker or has a higher surface tension.
- It explains why structures like vascular aneurysms and dilated bowel loops expand more easily the larger they get, and therefore why the diameter of an aneurysm is a useful indicator of how likely it is to rupture.
- Conversely, it explains why very small structures with a high surface tension are difficult to dilate – such as the alveoli of premature infants lacking surfactant. This is why the use of synthetic surfactant is so effective at reducing the work of breathing in these patients.

Conflict of interest: none.

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