Appendix: Quantum Physics Background

Modern physics' birth can be traced back to the reluctant introduction of what turned out to be a fundamental constant of nature, the so-called Planck constant, i.e., $h = 6.6 \times 10^{-34}$ *J.s*. Its introduction in an energy quantization formula was the only solution to the problem that emerged at the end of the 19th century when thermal emission and absorption spectra were experimentally analyzed for physical objects at various temperatures. Max Planck postulated (Planck, 1959) that these spectra originate from a discrete nature of energy levels in physical systems that can be quantized by the general relation between electromagnetic energy, *E*, and the corresponding frequency, *f (Hz)*:

$$E_n = (n+1/2)\, hf \qquad \text{(A1)}$$

where *n* enumerates the energy levels(dimensionless). This, rather an *ad hoc* assumption, led to a paradigm change in physics that departed from the laws of Newtonian (classical) physics and replaced them with wave function equations of quantum physics developed in the decades that followed. Consequently, Newton's equations of motion were replaced by non-relativistic Schrödinger's equations and relativistic Dirac's equations describing the system's wave functions' time evolution microscopic objects such as elementary particles (Fetter and Walecka, 2003). In both cases, the Planck constant, *h*, set both the time and energy scales due to the quantization relation.

Today, quantum theory is generally accepted as the most fundamental theory of matter, ultimately responsible for much of the last century's technological advancements. In this formalism, many of the variables taken as continuous by classical physics take on discrete values, hence the name quantum mechanics. Initially, the term quantum was used to denote a discrete packet of electromagnetic radiation. Quantum mechanics is a first-quantized or semi-classical physics theory (initially referred to as wave mechanics) in which particle properties are discrete, but field properties and interactions are continuous. Quantum field theory is a second-quantized theory in which all particle properties, field properties, and interactions are discrete except for the interactions with the field of gravity (Schweber, 2011). Quantum gravity is incomplete, and as yet, not integrated with the rest of quantum physics, a third-quantized theory in which gravity could also be made discrete (Rovelli, 2004).

In quantum physics, physical systems possess wave-like properties and particle-like properties, referred to as the principle of wave-particle duality or complementarity (Bohr, 1934, 1958). According to the Copenhagen interpretation of quantum mechanics (Griffiths, 2005), all the information about a particle or a system of particles can thus be described in a wave-like manner that is denoted mathematically by a wave function ϕ (x, t). The modulus squared of a particle's wave function describes the probability of finding this particle at a spatial location at an instant in time. Hence information about the particle's state is described probabilistically and not deterministically. However, wave functions behave similarly to waves such that they can be diffracted and can interfere together, forming superpositions. This implies that quantum particles may exist in multiple spatial locations and occupy any number of states at any moment in time. Once a measurement on the quantum system is performed, one of the multiple states is selected. The quantum superposition is reduced to a classical state in a process called the wave function collapse. The wave function of a system of mass m evolves by a so-called spreading effect such that its width $\Delta x(t)$ at time t can be linked to its original value $\Delta x(0)$ at time $t=0$ by a linear relationship:

$$\Delta x(t) = h\ t/2\pi\ m\ \Delta x(0) \qquad \text{(A2)}$$

This is interpreted as a continued delocalization of physical objects overtime or a loss of information about its position in space. Similarly, for the particle's wave function, it can be said that the product of its uncertainty in position $\Delta x(t)$ with its uncertainty in momentum $\Delta p(t)$ is always greater than $h/2\pi$, i.e., $\Delta x(t)\ \Delta p(t) > h/2\pi$. This inequality is referred to as the Heisenberg uncertainty principle.

The Heisenberg uncertainty principle can also be seen *via* another aspect of quantum theory. When two consecutive measurements are made on certain pairs of variables, called complementary variables (A and B), there is a fundamental limitation on the two measurements' precision. Thus, there is no state in which both complementary variables can be defined simultaneously with arbitrary accuracy. This property of quantum objects is a precise mathematical expression of the Heisenberg uncertainty principle, and it can be derived for any pair of non-commuting Hermitian operators A and B, representing physical observables, such that: $[A, B] = i\ (h/2\pi)$, which implies that (Griffiths, 2005)

$$<(A-<A>)^2> <(B-<B>)^2> > (1/2)\ |<[A,B]>| = (1/2)\ (h/2\pi) \qquad \text{(A3)}$$

where $[A, B] = AB - BA$ is a so-called commutator operation. Therefore, this is an empirical relationship and a fundamental property of quantum reality based on the properties of the algebra of Hermitian operators (Weinberg, 2013), used in quantum physics as mathematical representations of physically measurable variables (observables). As for manifestations of this inherent inability to precisely determine simultaneously the expectation values (denoted by $<\cdots>$) of any two conjugate physical observables (A and B), we find, for example, position and momentum as shown above, but also the angle and the corresponding angular momentum operator, two independent spin component operators and, very importantly, energy and time such that the uncertainties in the latter two variables fulfill the inequality: $\Delta E\ \Delta t > h/2\pi$.

Reference

Bohr, N. (1934) Atomic theory and the description of nature. Cambridge: Cambridge University Press.

Bohr, N. and Noll, W. (1958) Atomic physics and human knowledge. American Journal of Physics 26, 596-597.

Fetter, A. L., and Walecka, J. D. (2003) Quantum theory of many-particle systems. Courier Corporation.

Griffiths, D. J. and Schroeter, D. F. (2005) Introduction to Quantum Mechanics. Pearson Education India.
Planck, M. (1959) The Theory of Heat Radiation. Courier Corporation.
Rovelli, C. (2004) Quantum Gravity. Cambridge: Cambridge University Press.
Schweber, S. S. (2011) An introduction to relativistic quantum field theory. Courier Corporation.
Weinberg, S. (2013) Lectures on Quantum Mechanics. Cambridge: Cambridge University Press.